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Nonlocal validity of an asymptotic one-dimensional nematicon solution

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Abstract
The propagation of coherent, polarized light in a nematic liquid crystal, governed by the nematicon equations, is considered. It is found that in the special case of 1 + 1 dimensions and the highly nonlocal limit, the nematicon equations have an asymptotic bulk solitary wave solution, termed a nematicon, which is given in terms of Bessel functions. This asymptotic solution gives both the ground state and the symmetric and antisymmetric excited states, which have multiple peaks. Numerical simulations of nematicon evolution, for parameters corresponding to experimental scenarios, are presented. It is found, for experimentally reasonable parameter choices, that the validity of the nonlocal approximation depends on the type of nematicon, as in some cases the asymptotic nematicon undergoes large amplitude oscillations. The magnitude of the nonlocality parameter for the asymptotic nematicon amplitude to be constant over a typical experimental propagation distance is also determined.

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1. Introduction

Spatial solitary waves in bulk media result from a balance between the diffractive spreading of a light beam and nonlinear and/or nonlocal focusing. Such solitary waves have generated much interest due to their possible applications as re-configurable 'circuits' for all-optical information processing [1, 2].

One particular nonlinear, nonlocal optical medium which has received much attention is a nematic liquid crystal, due in part to its large nonlinear response which allows nonlinear effects to be observed over small (∼mm) distances. A series of elegant experiments have shown that stable spatial solitary waves, so-called nematicons, can propagate in nematic liquid crystals [3, 4].

The equations governing nematicon propagation are a coupled system of two nonlinear partial differential equations in 2 + 1 dimensions and as such are difficult to solve, with no known exact solutions. For this reason most existing theoretical work has been numerical [5–8] or based on using a mix of various asymptotic, approximate and numerical methods [9–12]. In parallel with this work on nematicons, there has been general research on solitary waves for nonlocal, nonlinear Schrödinger (NLS) equations,

$$\mathrm{i}\frac{\partial u}{\partial z} + \frac{1}{2}\nabla^2 u + u\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} G(x-x', y-y')|u(x', y', z)|^2\,\mathrm{d}x'\,\mathrm{d}y' = 0, \qquad (1)$$

for various kernels G [9, 13, 14]. The particular case of nematicons corresponds to a kernel G given in terms of modified Bessel functions, which results in solutions of the nonlocal NLS equation (1) being difficult to find. As a consequence most of the analysis of this nonlocal NLS equation has been done for simplified kernels, such as Gaussians and exponentials [9, 13, 14].

In this paper the nematicon equations in 1 + 1 dimensions will be considered. It will be shown that these nematicon equations reduce to a one-dimensional form of the nonlocal NLS equation (1) with an exponential kernel. In the limit of large nonlocality this equation possesses an asymptotic nematicon solution in terms of Bessel functions, which gives both the ground-state solitary wave, which has a single peak, and the symmetric and antisymmetric excited states, which have multiple peaks. In a different physical situation, the governing partial differential equations for quadratic solitons in a $\chi^{(2)}$ material are different to those for nematic liquid crystals, but their solitary wave solution is the same, see [15]. However the validity of the asymptotic solitary wave as a solution of the full equations was not investigated in Nikolov *et al* [15]. In the present work this validity is investigated by using the asymptotic nematicon solution as an initial condition for numerical solutions of the full nematicon equations. These numerical results show that both the ground- and excited-state nematicons are stable in the highly nonlocal limit, in contrast to local media for which the excited-state solitary waves are unstable [1]. However the asymptotic assumption used to derive this solution means that it is steady only up to a value of z which increases with the degree of nonlocality, after which its amplitude oscillates to a new steady state. Moreover, for realistic experimental scenarios, the value of z for which the nematicon is steady is much shorter than the length of the liquid crystal cell.

2. Governing equations

Let us consider coherent, polarized light propagating in the z direction in a liquid crystal cell, with the (x, y) coordinates orthogonal to this propagation direction and the light polarized in the x-direction. In addition, a static electric field is applied in the x-direction so that in the absence of light the nematic director makes an angle $\hat{\theta}$ to the z-direction. We then set θ to be the perturbation of the director angle from this pre-tilt angle due to the light and E to be the electric field envelope of the light. The equations governing the propagation of light through the liquid crystal cell are then

$$\mathrm{i}\frac{\partial E}{\partial z} + \frac{1}{2}\nabla^2 E + \sin 2\theta\, E = 0, \qquad \nu\nabla^2\theta - q\sin 2\theta = -2|E|^2\cos 2\theta, \qquad (2)$$

where the Laplacian ∇^2 is in the (x, y) plane and z is the direction of propagation of the light [2–5, 11, 12]. The parameter ν measures the elasticity of the nematic and q is related to the energy of the static electric field which pre-tilts the nematic [4, 10, 16].

Peccianti *et al* [17] experimentally considered the steering of nematicons within a three-dimensional liquid crystal cell. They found that they could steer the nematicons by the application of a voltage which changes the elevation angle, η_0, of the nematic liquid crystal molecules. In this experimental scenario the light propagates in the z-direction and the voltage is applied in the x-direction. The elevation angle η_0 is the angle that the nematic molecules make with the y–z plane. When the voltage is zero the molecules lie in the y–z plane and the elevation angle $\eta_0 = 0$. In this limit ν is large and the nonlocal approximation is valid. As η_0 approaches 90° however, and the nematic molecules lie in the x–y plane, ν becomes small and the governing equations (2) reduce to a saturating NLS equation, see [10, 11]. Moreover, Rasmussen *et al* [13] considered the experimental nematic liquid crystal cell of Conti *et al* [4] used for the propagation of optical solitons. They found that the value of $\nu/q \approx 25$ for this experimental work.

In this work we consider the nonlocal limit, for which ν is large. In this case it can be seen from the director equation, the second of (2), that θ is small. In this large-ν limit the response of the director to the electric field is nonlocal due to the slow decay of the crystal distortion produced by the light. So in the nonlocal limit the nematicon equations (2) can be approximated by

$$\mathrm{i}\frac{\partial E}{\partial z} + \frac{1}{2}\nabla^2 E + 2\theta E = 0, \qquad \nu\nabla^2\theta - 2q\theta = -2|E|^2. \tag{3}$$

We now consider the nonlocal nematicon equations (3) for one transverse dimension x, so that the nematicon is one dimensional and independent of the variable y. In this case the director equation, the second of (3), can be solved using Green's functions and the result substituted into the first of (3), resulting in the nonlocal NLS equation

$$\mathrm{i}E_z + \frac{1}{2}E_{xx} + \frac{\gamma E}{2q}\int_{-\infty}^{\infty} \mathrm{e}^{-\gamma|x-x'|}|E(x', z)|^2\,\mathrm{d}x' = 0, \tag{4}$$

where $\gamma = (2q/\nu)^{1/2}$. In the nonlocal limit, ν is large, so that γ is small. This means that the director response, given by the exponential kernel in the nonlocal NLS equation (4), is much wider than the nematicon in the electric field. As in Briedis *et al* [18] the nonlocal integral can then be approximated to give the final equation

$$\mathrm{i}\frac{\partial E}{\partial z} + \frac{1}{2}\frac{\partial^2 E}{\partial x^2} + \frac{\gamma}{2q}\,\mathrm{e}^{-\gamma|x|}PE = 0, \tag{5}$$

where

$$\theta = \frac{1}{\sqrt{8q\nu}}P\,\mathrm{e}^{-\gamma|x|}, \qquad P = \int_{-\infty}^{\infty}|E(x', z)|^2\,\mathrm{d}x', \tag{6}$$

valid for $\nu \gg 1$, $\gamma \ll 1$. The quantity P is the total power per unit length of the nematicon (as we are considering the one-dimensional case) and is a conserved quantity.

The approximate equation (5) has the solitary wave (nematicon) solution

$$E = f(x)\,\mathrm{e}^{\mathrm{i}\sigma z}, \qquad f = AJ_n(\lambda\,\mathrm{e}^{-\gamma x/2}), \tag{7}$$

where $n = 2\sqrt{\sigma\nu}/\sqrt{q}$ and $\lambda^2 = 2^{3/2}P\sqrt{\nu}/q^{3/2}$ for $x \geqslant 0$ and the symmetric solution for $x \leqslant 0$. Here A is a constant and J_n is the Bessel function of order n.

The definition (6) for the power P gives

$$A^2 = \frac{\gamma P}{4}\left[\int_0^{\lambda}\frac{1}{\psi}J_n^2(\psi)\,\mathrm{d}\psi\right]^{-1}, \tag{8}$$

on using symmetry. Now the nematicon solution (7) decays to zero as $x \to \infty$, since $n > 0$. The free parameter of the nematicon solution (7) is the propagation constant σ. This asymptotic

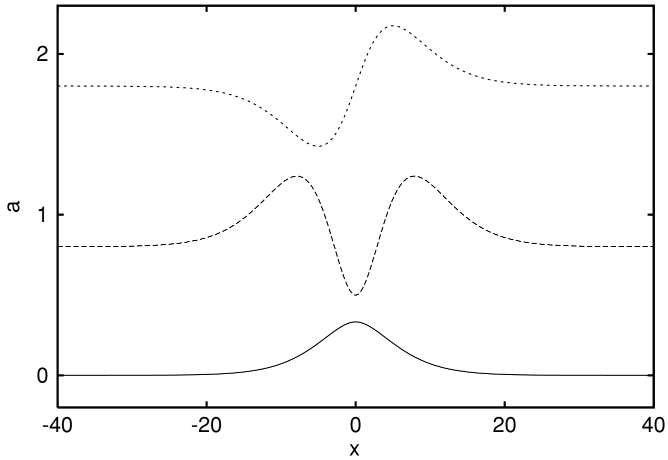

Figure 1. The ground-state nematicon (solid line) and the first symmetric (long dashes) and antisymmetric (short dashes) excited states for $\nu = 100, q = 1, \sigma = 4/100, n = 4$. Shown is the amplitude $a = |E|$ versus x. The symmetric and antisymmetric excited states have been translated vertically by 0.8 and 1.8, respectively.

nematicon solution is similar to an asymptotic solution of the equations for a quadratic soliton in a $\chi^{(2)}$ material, see [15], but the governing partial differential equations are different.

To obtain symmetric nematicons we apply the condition $f' = 0$ at $x = 0$, which gives $\lambda = j'_{n,m}$, where $j'_{n,m}$ is the mth zero derivative point of $J_n(x)$, see [19]. The smallest zero derivative point $j'_{n,1}$ corresponds to the ground-state nematicon which decays monotonically as $|x| \to \infty$. The nematicons corresponding to the higher zero derivative points $j'_{n,m}$, $m > 1$, are excited states, with the nematicon corresponding to $j'_{n,m}$ having an odd number $(2m - 1)$ of peaks [1]. To obtain antisymmetric nematicons we apply the condition $f = 0$ at $x = 0$, which gives $\lambda = j_{n,m}$, $m \geqslant 1$, where $j_{n,m}$ is the mth zero of $J_n(x)$. The antisymmetric solution is taken for $x \leqslant 0$ in (7). These nematicons have an even $(2m)$ number of peaks.

Figure 1 shows examples of the ground-state nematicon, and the first symmetric and antisymmetric excited states. The ground-state nematicon has power $P = 1$, while $P = 2.04$ and 3.05 for the antisymmetric and symmetric exited states, respectively. The excited states are broader, in the transverse direction, than the ground-state nematicon, with the symmetric excited state being slightly broader than the antisymmetric one.

3. Numerical solutions

The nematicon (7) is the solution of the nonlocal reduction (5) of the full nematicon equations (2). Two questions then arise: (i) how accurate is this solution as an approximation to the nematicon solution of the full equations and (ii) is this nematicon solution stable, particularly the excited states? Reference [15] did not fully address this issue as they only compared numerically obtained quadratic soliton profiles with asymptotic profiles, finding that the ground-state asymptotic profile differed significantly from the numerical profile around the peak. These questions will be answered here by using the nematicon solution (7) as an initial condition for the full nematicon equations (2).

Peccianti *et al* [17] provided parameter values for the full range of elevation angles, η_0, allowing values of the non-dimensional parameters of (2) to be estimated for realistic

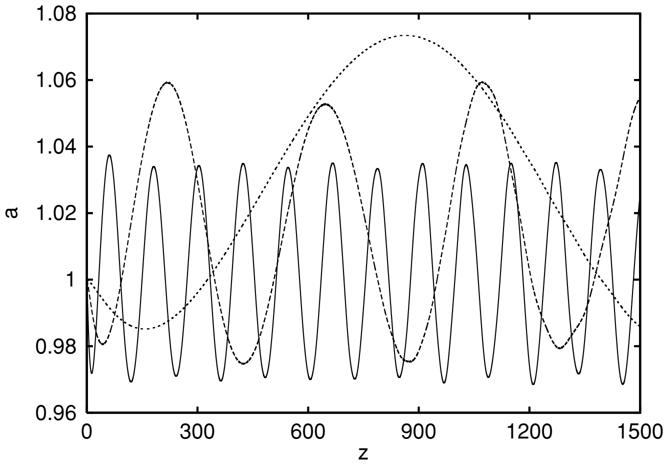

Figure 2. Evolution of ground-state nematicons according to the full governing equations (2). Shown is the scaled amplitude $a = |E(0, z)|/|E(0, 0)|$ at $x = 0$, for $\nu = 100$ (solid line), $\nu = 1000$ (long dashes) and $\nu = 10\,000$ (short dashes).

experimental scenarios. For example, using the experimental data from Peccianti *et al* [17] gives $\nu/q \approx 2000$ when $\eta_0 = 45^{\circ}$ and $\nu/q \approx 50$ when $\eta_0 = 80^{\circ}$. Also, $\nu/q \approx 25$ for the experiments of Conti *et al* [4], so that ν/q is not always large in experimental scenarios. Moreover, the non-dimensional propagation distance of the nematicon (the length of the liquid crystal cell) in Peccianti *et al* [17] was $z \approx 500$.

Hence, for our numerical experiments we set $q = 1$ and consider the nonlocal approximation as ν varies over three orders of magnitude, $\nu = 100$, 1000 and 10 000. As seen above these values of ν will all occur as the elevation angle is varied [17]. Moreover, we wish to determine nematicon stability over a experimentally realistic length scale, z up to 500.

The numerical solutions were found using the Dufort–Frankel finite difference scheme to solve the electric field equation, the first of (2). For the director equation, the second of (2), Gauss–Seidel iteration was used with successive over relaxation. An advantage of the Dufort–Frankel and Gauss–Seidel schemes is that they are both explicit methods with low storage costs. The step sizes used were $\Delta x = 0.4$ and $\Delta z = 2 \times 10^{-3}$. Note that $\Delta z/\Delta x$ must be small to ensure consistency of the Dufort–Frankel finite difference scheme.

Figure 2 shows the evolution of the ground-state nematicon solution (7) (using $j'_{n,1}$) for $\nu = 100$, 1000 and 10 000, as given by numerical solutions of the full nematicon equations (2). All the nematicons have power $P = 1$ with $q = 1$. Plotted is the scaled electric field amplitude $a = |E(0, z)|/|E(0, 0)|$ at the origin $x = 0$. As a test the evolution of the nematicon solution using the nonlocal approximation (5) was also calculated. This showed that the ground-state nematicon solution is a steady solution of (5), with the numerical amplitude a remaining constant (to the level of machine roundoff error) to $z = 500$. A curve showing the evolution of the nematicon amplitude using the nonlocal approximation (5) is not included in figure 2 as it is merely a horizontal line with $a = 1$.

The figure shows that the initial pulse oscillates in a near periodic manner as it relaxes to a steady-state nematicon of the full governing equations on an extremely long z-scale. For this example the amplitude initially decreases. This is due to the amplitude of the initial condition being less than that of the final steady nematicon. If the initial amplitude is greater than that of the final steady nematicon, then the amplitude of the pulse initially increases.

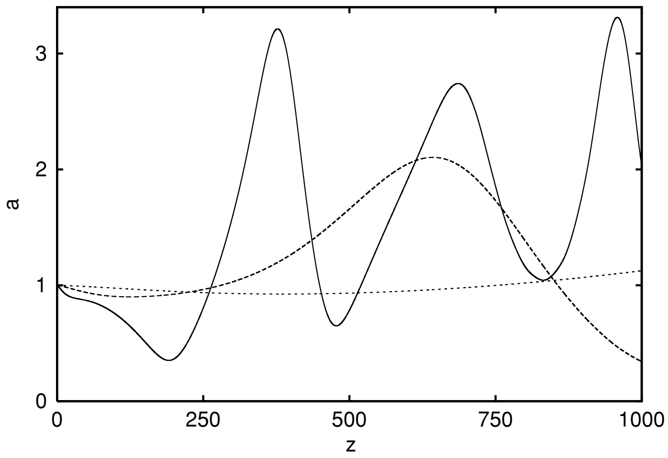

Figure 3. Evolution of first symmetric excited states according to the full governing equations (2). Shown is the scaled amplitude $a = |E(0,z)|/|E(0,0)|$ at $x = 0$, for $\nu = 100$ (solid line), $\nu = 1000$ (long dashes) and $\nu = 10\,000$ (short dashes).

The amplitude of the nematicon oscillates with an amplitude of about 0.07 for all three values of ν. The wavelength of the oscillations increases as ν increases, from 107, 382 to 1472. The amplitude of the final steady nematicon solution of the full equations evolving from the asymptotic nematicon then differs by about 4% in each case. The ground-state asymptotic nematicon is then a reasonable approximation to the nematicon solution of the full equations for realistic values of the nonlocality parameter ν.

However in experimental situations, as the nematicon equations (2) have been non-dimensionalized with respect to the Rayleigh length, the non-dimensional propagation length z in figure 2 at which the nematicon solution starts to evolve is far shorter than the length of a liquid crystal cell (which is about $z = 500$). So for experimental situations the ground-state nematicon (7) is not a fully steady solution (with less than 1% variation in amplitude) of the full nematicon equations (2), unless $\nu \approx 150\,000$.

Figure 3 shows the evolution of the first symmetric excited state of (7) (using $j'_{n,2}$) for $\nu = 100, 1000$ and $10\,000$, as given by the numerical solution of the full nematicon equations (2). All the nematicons have constant power $P = 1$ with $q = 1$. Shown is the scaled electric field amplitude $a = |E(0,z)|/|E(0,0)|$ at the origin. Again the nematicon solution is a steady solution of the nonlocal approximation (5). The oscillations of the first excited state are of much larger amplitude compared with the corresponding ground-state nematicons. For example, the peak to trough oscillation amplitudes are 2.9, 1.8 and 0.82 as ν increases. The wavelengths of the oscillations increase as ν increases and are approximately twice the magnitude of the wavelengths of the corresponding ground-state nematicon oscillations.

For the symmetric excited state ν would need to be greater than 350 000 for the nematicon amplitude to be steady (less than 1% variation) up to $z = 500$. Clearly the symmetric excited states are evolving to a steady state which is far from the initial asymptotic nematicon state. The nonlocal approximation leading to equation (5) is then clearly not valid for experimental values of ν when excited-state nematicons are considered.

Figure 4 shows the evolution of the first antisymmetric excited state of (7) (using $j_{n,1}$) for $\nu = 100, 1000$ and $10\,000$, as given by the numerical solution of the full nematicon equations (2). Again all the nematicons have constant power $P = 1$ with $q = 1$. Shown is the scaled electric field amplitude a at the nematicon peak. The figure shows that the amplitudes

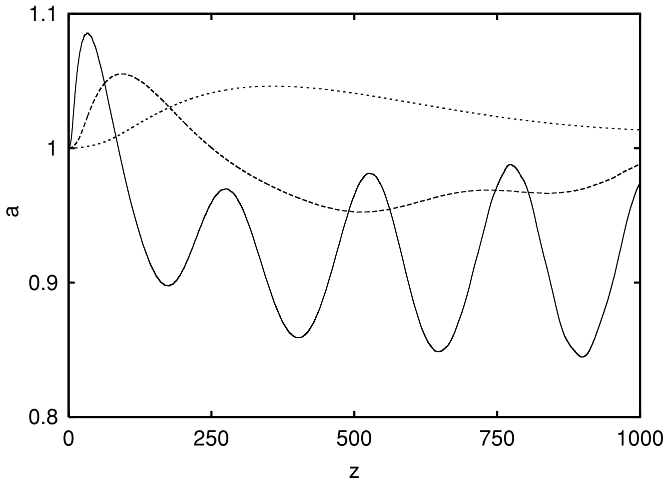

Figure 4. Evolution of first antisymmetric excited states according to the full governing equations (2). Shown is the scaled amplitude $a = |E(0,z)|/|E(0,0)|$ at the nematicon peak for $\nu = 100$ (solid line), $\nu = 1000$ (long dashes) and $\nu = 10\,000$ (short dashes).

of the nematicons increase initially due to the amplitudes of the initial conditions being greater than that of the final, steady, exact solutions.

The amplitude oscillations of the first antisymmetric excited state are slightly larger than those of the ground state, but much smaller than those of the first symmetric excited state. The peak to trough oscillation amplitudes are 0.19, 0.10 and 0.03 as ν increases. The wavelengths of the oscillations are approximately the same as those of the ground-state nematicon amplitude oscillations. For this antisymmetric excited state ν would need to be greater than 160 000 for the nematicon amplitude to be steady (with less than 1% variation). These numerical simulations show that for this antisymmetric excited state the nonlocal approximation is reasonable.

For local NLS-type equations the ground-state solitary wave solutions in (1+1) dimensions are stable (but unstable in (2 + 1) dimensions), while the excited-state solitary waves in (1 + 1) dimensions are unstable, see [1]. In the case of the nematicon equations, it is the nonlocal nature of the nonlinearity which causes the excited states to be stable.

The basic issue is the accuracy of the nonlocal approximation by which (4) is approximated by (5). These equations differ in the nonlinear terms

$$\frac{\gamma}{2q} P\,\mathrm{e}^{-\gamma|x|}, \qquad \frac{\gamma}{2q}\int_{-\infty}^{\infty} \mathrm{e}^{-\gamma|x-x'|}|E(x',z)|^2\,\mathrm{d}x', \tag{9}$$

multiplying E, which we shall term the potential $U(x)$ due to the analogy with the linear Schrödinger equation of quantum mechanics. Figure 5 shows the difference between the scaled potentials (9) for the ground-state and first symmetric and antisymmetric excited states. The scaled potentials shown are for the nematicons from figures 2–4, for $\nu = 10\,000$. If the nonlocal approximation (5) were exact, then all of (9) would be the same. The figure shows that the nonlocal approximation is excellent for the ground-state nematicon away from its peak, with the difference in potentials at $x = 0$ being 16%. For the excited symmetric and antisymmetric states the nonlocal approximation is less accurate, with significant variation over the whole range of x, with differences at $x = 0$ of 47% and 34% respectively. Qualitatively this is due to the excited-state nematicons being broader in the transverse direction than the

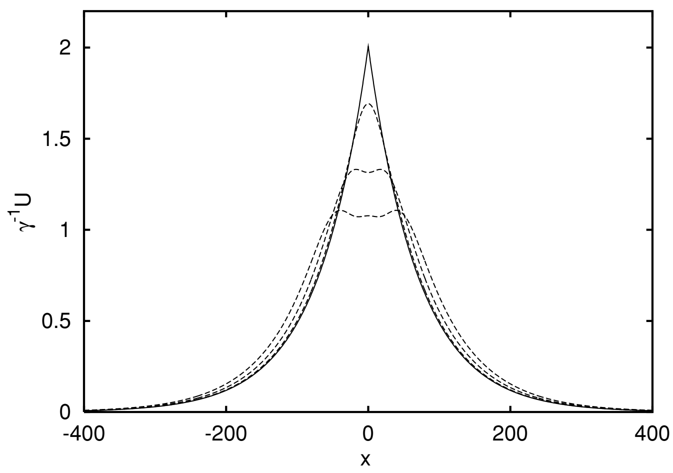

Figure 5. The scaled potential $\gamma^{-1}U$ versus x. First of (9) (solid curve); second of (9) for the ground (highest dashed curve at $x = 0$), and the first symmetric (lowest dashed curve at $x = 0$) and first antisymmetric excited states (middle dashed curve at $x = 0$). Here $\nu = 10\,000$ and $P = 1$.

ground states. The greater difference in the potential for the excited state explains why the nonlocal approximation is not good for excited-state nematicons for experimental values of ν. It also explains why the nonlocal approximation is better for the first antisymmetric nematicon than for the corresponding first symmetric case.

For the other two ground-state nematicons shown in figure 2 the differences in potential at $x = 0$ are 32% and 23% for the $\nu = 100$ and $\nu = 1000$ cases, respectively. Hence the magnitude of this difference in potential at $x = 0$ provides a convenient measure of the accuracy of the nonlocal approximation, as the smaller the magnitude, the larger the z value for which the asymptotic nematicon solution is steady.

4. Conclusions

The accuracy of one-dimensional solitary wave (nematicon) ground and symmetric and antisymmetric excited states of a nonlocal approximation to the nematicon equations for guided wave propagation in nematic liquid crystals in the highly nonlocal limit has been examined. Numerical solutions show that both the asymptotic ground, symmetric and antisymmetric excited states are stable solutions of the full nematicon equations. However numerical solutions further show that the asymptotic symmetric excited state is not a good approximation to the solution of the full nematicon equations for experimental values of the nonlocality as it undergoes the typical oscillatory evolution to an exact nematicon solution of the full nematicon equations which is far from the asymptotic solution.

For the ground and first antisymmetric excited states it was found that the nonlocal approximation is reasonable for experimental parameter choices. However, even for these cases, the value of ν must be extremely large (>160 000) for the nematicon amplitude to be constant for experimental values of the propagation distance z.

The coherent light beams used in experiments usually show circular symmetry. Unfortunately the one-dimensional nematicon solution of the present work cannot be extended to this circularly symmetric, two-dimensional case.

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References

[1] Kivshar Y S and Agrawal G P 2003 *Optical Solitons. From Fibers to Photonic Crystals* (San Diego, CA: Academic)
[2] Assanto G, Peccianti M and Conti C 2003 Nematicons: optical spatial solitons in nematic liquid crystals *Opt. Photon. News* **14** 44–8
[3] Conti C, Peccianti M and Assanto G 2003 Route to nonlocality and observation of accessible solitons *Phys. Rev. Lett.* **91** 073901
[4] Conti C, Peccianti M and Assanto G 2004 Observation of optical spatial solitons in a highly nonlocal medium *Phys. Rev. Lett.* **92** 113902
[5] Assanto G and Peccianti M 2003 Spatial solitons in nematic liquid crystals *IEEE J. Quantum Electron.* **39** 13–21
[6] Assanto G, Peccianti M, Brzdakiewicz K A, De Luca A and Umeton C 2003 Nonlinear wave propagation and spatial solitons in nematic liquid crystals *J. Nonlinear Opt. Phys. Mater.* **12** 123–34
[7] Fratalocchi A, Asquini R and Assanto G 2005 Integrated electro-optic switch in liquid crystals *Opt. Express* **13** 32–7
[8] Fratalocchi K A, Brzdakiewicz A, Karpierz M A and Assanto G 2005 Discrete light propagation and self-trapping in liquid crystals *Opt. Express* **13** 1808–15
[9] Yakimenko A I, Lashkin V M and Prikhodko O O 2006 Dynamics of two-dimensional coherent structures in nonlocal nonlinear media *Phys. Rev.* E **73** 066605
[10] Garcia-Reimbert C, Minzoni A A and Smyth N F 2006 Spatial soliton evolution in nematic liquid crystals in the nonlinear local regime *J. Opt. Soc. Am.* B **23** 294–301
[11] Garcia-Reimbert C, Minzoni A A, Smyth N F and Worthy A L 2006 Large-amplitude nematicon propagation in a liquid crystal with local response *J. Opt. Soc. Am.* B **23** 2551–8
[12] Minzoni A A, Smyth N F and Worthy A L 2007 Modulation solutions for nematicon propagation in non-local liquid crystals *J. Opt. Soc. Am.* B **24** 1549–56
[13] Rasmussen P D, Bang O and Krolikowski W 2005 Theory of nonlocal soliton interaction in nematic liquid crystals *Phys. Rev.* E **72** 066611
[14] Lopez-Aguayo S, Desyatnikov A S, Kivshar Y S, Skupin S, Krolikowski W and Bang O 2006 Stable rotating dipole solitons in nonlocal optical media *Opt. Lett.* **31** 1100–2
[15] Nikolov N I, Neshev D, Bang O and Krolikowski W 2003 Quadratic solitons as nonlocal solitons *Phys. Rev.* E **68** 036614
[16] Garcia-Reimbert C, Hume G, Minzoni A A and Smyth N F 2002 Active tm mode envelope soliton propagation in a nonlinear nematic waveguide *Physica* D **167** 136–52
[17] Peccianti M, Conti C, Assanto G, De Luca A and Umeton C 2004 Routing of anisotropic spatial solitons and modulational instability in liquid crystals *Nature* **432** 733–7
[18] Briedis D, Petersen D E, Edmundson D, Krolikowski W and Bang O 2005 Ring vortex solitons in nonlocal nonlinear media *Opt. Express* **13** 435–43
[19] Abramowitz M and Stegun I A 1965 *Handbook of Mathematical Functions* (New York: Dover)